The Many Faces of Zero, One & Two As Mirrored in the Looking Glass of Bernhard Riemann's Imaginary Landscape Revealing The Pattern of The Prime Numbers.

Human beings have an innate number sense and it developed culturally similarly to the way it develops in children. We learn to order objects and groups according to size, what is larger compared to what is smaller, before we can count them. Studies with four-year-olds show that as they grasp learning the natural numbers, some have trouble with the concept of zero. When children were given a test of squares with a different number of dots in each about 75% correctly identified the square with the fewest number of dots. But when an empty square with no dots was given as an option, only about half of the children get the right answer. Does the square with no dots count as a box with the fewest dots? Zero is primitive but the concept evolved rather late in the construction of human number theory and long after the first use of numbers from the Stone Age. The Greek with all their sophistication never savvy in early mathematics never isolated the number zero as a foundational concept. The Babylonians in the Near East and the Mayans in America had symbols for zero but they thought it only as a functioning placeholder. We take it for granted now but zero as a placeholder was a nifty innovation for calculating but zero as a true number was a leap forward in progressive thinking—from nothing to something countable and subject to arithmetic manipulation. But zero is not just an abyss of nothingness, an infinity of emptiness but rather a reified link that connects the abstract to the concrete in a defined sphere or realm. For an empty fruit bowl on a dining room table could imply an emptiness of anything and everything, zero also implies a vacuum within a delineated perimeter of presence----as the tabletop, empty fruit bowl has 'zero' fruit. Zero must have a defining entity whether it is apples or as in arithmetic the number line. A Universe devoid of everything contains nothing; but unlike the concept of zero it in its all-encompassing emptiness it offers no comparison that infers the magnitude of what is missing; it is this contradictory juxtaposition of what is there and what isn't that encapsulates zero. A donut hole is a zero because it is surrounded by the dough of the fried confectionary. When *all* is *everything* or when *all* is *nothingness*, one has contrarian worlds where neither opposing feature truly entertains the definition of zero. Meaning, a total nihilistic cosmos despite containing nothingness, does not reflect the identity of zero nor does it capture its multifaceted if often ambiguous nature. Dating back to the 5th century, Indian mathematicians defined 'nothing' as possessing both 'something and nothing, presence and absence, positive and negative. It is *nothing* in a *particular place*; zero in quantifying the lack of anything within a defined niche or space, belies its first use simply as a mere placeholder in calculations and not its acceptance as a legitimate integer. It is all three: a placeholder especially marking a starting point or origin, a concept denoting the abyss and a number.

It took time for the concept of a true zero to spread from India to the rest of Asia, into Africa and Europe but merchants and traders from Arabia brought the new number and its utility and eventually also the change to Indo-Arabic numerals; the transformation from Roman numerals and other archaic counting systems came slowly despite the Indo-Arabic numbers apparent facility to make the bookkeeping of international trading easier. Italian mathematician Leonardo Fibonacci brought zero to Europe in the 12th century after he learned of it while with his father in Algeria who worked assisting Italian traders in the seaport of Bugia. He used zero only as a placeholder for numerating the decimal system in his published *The Book of Calculation* that he authored when he returned to Europe but his work did little at first to establish zero. Fibonacci translated the word *sifr* from Arabic to the Latin word *zephyrum*, meaning emptiness; it was the Venetians who later introduced the term *zero* within their vernacular. It was originally the ninth-century Arabian polymath Al-Khwarizmi whose own novel mathematical tome revealed the usefulness of a true zero as an integral part of a number system for counting and arithmetic. Have you ever tried to multiply using Roman numerals? Most elementary students do, if only to illustrate how cumbersome it is compared to 0, 1, 2, 3, 4, 5, 6, 7, 8, 9 and its endless numerical combinations.

There are an infinite number of varied sizes of infinity as established by the German mathematician Georg Cantor. For example, there is a magnitude difference between the set of infinity represented by all positive integers versus the larger set of all real numbers even though bought subsets are counted into infinity and beyond. Zero also comes in different sizes although this is not emphasized or often illustrated. Zero is specific and as such in a poor season no apples hanging on one lone apple tree is less than the size of the zero derived from an orchard that fails to produce fruit. The arithmetic operation that divides [0] zero by a divisor to equal zero only is logical if only zero itself can be partitioned into smaller segments or aliquots. It is more accurate to define 0/0 as indeterminate because one can't divide nonexistent numerators and denominators. Ambiguities with zero in arithmetic are evident but are often glossed over. Zero divided by zero is considered by many to be zero but why isn't it one, since any other number divided by itself is one, at least it should be considered as one times the entity zero. No? In fact, any other number divided by zero is considered indeterminate. Any number multiplied by zero is considered zero but might not one argue that 1, 000, 000 times zero is a larger nullity than zero-times-zero or one-times-zero especially if the parameters to define the scope of the original zero are exactly equivalent. No parasites on a guppy is perhaps a smaller zero than no similar infestation on a blue whale. Or consider the Latin phrase *E pluribus unum*—from *many, one*—does this have ramifications for the varying sizes of zero that may be welded together into one continuum of a larger nothingness? Also, zero times zero is indeterminate and contradictory because multiplication isn't logically operable when there are no terms to be multiplied so in truth its equivalent to the indeterminacy of dividing a number by zero. Division is at times better defined as repetitive subtraction as exemplified by 10/2 which equates two being subtracted five times from ten before reaching zero. Okay, then why isn't ten divided by zero represented by a quotient of infinity because one can subtract zero from ten forever without decreasing it at all never mind extinguishing it to nothing. Addition and subtraction of zero gives a logical answer with no change in number but multiplication and division answers are dubious and ambiguous. Is 5 X 0 representative of adding five zeros [i.e.. 00000]---a larger realm of zero---or does it just eviscerate the number five [0 x 5 =0]?. No arithmetic operation functions legitimately if not on 'something.' ----can one really talk of raising zero to a power or deriving its root? The number zero in the number line works because it is a relative comparison of nothingness compared to the other numbers of positive or negative value on that same number line. One cannot pitch and hit without a baseball to be pitched and batted in a baseball game.

In a sense though one can multiply by zero by comparing its valueless proportionally with the selected metrics being measured. Take our number line or the demarcations on a measuring ruler. The demarcation from zero or the starting edge of the ruler denotes a different relative value of nothing depending on what each delineated segment represents. If each millimeter mark represents but one atom of gold even after a good distance one does not have more than a minute amount of precious metal. If each demarcation denotes a Fort Knox worth of gold bullion then in only a few millimeters of distance, the accumulation would become a windfall of riches. Zero is the riverbed whose currents flow according to the differential of its riverbanks—slow and lazy moving waters through a broad marshy river delta or swift and ferocious torrents as the flooded Colorado River flows through the chiseled sandstone walls of the mighty Grand Canyon. In a peculiar sense, you can multiply or divide zero by multiplying the magnitude of what lays beside it; if one multiplies every positive number on the number line by one hundred doesn't that divide the proportional magnitude of zero by the same. And vice versa, when one divides all positive numbers by one hundred doesn't that magnify the value of zero by 100 in comparison? The true significance of zero is the differentiation that it represents at a specific place, for 1<u>0</u>1,222, let's say, has a zero that means there are no 10, 000 units whereas 1<u>0</u> has a zero that symbolizes no single digits.

There is no greater evidence of the morphic shape and different faces of zero and the importance of their location than as evidenced in the Riemann Hypothesis which has three types of zeros: the zero at the intersection of the horizontal axis of the real number line and the vertical line of imaginary numbers, the trivial zeros situated on the horizontal x-axis and the nontrivial zeros that Bernhard Riemann proposed all fell on the ley line that runs upward parallel to the center vertical axis [y] of imaginary numbers and intersects the real number line horizontal axis [x] at positive one-half [1/2]. Riemann managed to find a connection between prime numbers and the nontrivial zeros, both infinite in their scale, and he discovered that these nadir points of the zeta-function landscape were not randomly scattered but he intuited that all of these insightful zeros would lie squarely on this mysterious magical *critical* line that intersected the horizontal x-axis at one-half. That the *critical* ley line is at ½ is intriguing from an analysis of 0 and 1and the indeterministic nature of the results from the simple, first steps arithmetic manipulations performed on them. Recalling 1/0, 0/0 and 0/1 are arguably indeterministic or ambiguous in their proposed answers but the result is concrete at 1/2 when one divides the first whole integer one by its neighbor two. There is no equivocation that one divided by two is one-half and it is equidistant from the zero of the origin and the very cliff edge that contain the height of the zeta-function output that intersects the x-axis at number one. At one-half, the zeta-function landscape bottoms out to sea level at the exact coordinates of the nontrivial zeros; it is at these nadir points that a mathematical cartographer if he knows the exact coordinate points of the zeros could reconstruct everything about the entire zeta-function landscape. The critical ley-line acts as a seam where at the zero points on the vertical imaginary axis at one-half tells you everything about the unfolding topography of the landscape It is like just knowing the topography of the city of Denver but that alone gives one a way to visualize an accurate topographic map of the entire state of Colorado and its Rocky Mountains. For us, we must ponder why this omniscience vista point defined by the nonrandom distribution of the nontrivial zeros that allowed Riemann to accurately estimate the number of primes up to a chosen number [N] lies at the first result among the primordial components of the natural numbers where indeterminate ambiguity evaporates when the first integer is divided by the second. Riemann's revolutionary work connected the appearance of random primes with a regimented front of orderly nontrivial zeros by feeding imaginary numbers into the zeta-function and the ordered arrangement of the zeros mirrored the primes themselves and the density of one is the reciprocal of the other. The primes occur further apart as one counts toward infinity while the zeta zeros get closer together as one ascends the vertical strip along the ley line anchored at its intersection base on the horizontal real number axis at **one-half.** This critical line splits the critical zone that lies from 0 to 1 and consists of all complex numbers with their real part equal to one-half. The number **one**, also has a magical importance in the transformational landscape,-----a mathematical Alice-in-Wonderland world with shadows and topographical relief that is constructed by feeding the coordinates of imaginary numbers into the zeta-function and as the output is recorded a two-dimensional map of imaginary numbers is expanded into a towering graphic structure of four dimensions. Riemann's inspirational work showed that to understand how the prime numbers emerge along the one-dimensional real number line requires a new comprehension of his zeta function in the complex plane: a two-dimensional world depicted by a Cartesian graph depicting a real number axis [x] and imaginary number axis [y]. There are two dimensions to keep track of the coordinates of the imaginary numbers being fed into the zeta-function, the third and fourth dimensions are built up from marking the two coordinates describing the imaginary output spit out by the function. Looking eastward, the zeta-landscape levels out into a level plain one unit high above the zero-sea level. There exist a ridge of hills as one plots the zeta-function output coordinates that run north-to-south and intersects the east-west axis at the number **one.** Furthermore, strikingly there exists at the intersection at the number 1 a peak that rises precipitously high all the way to infinity. This summit represents when the number one is fed into the zeta-function where the output reaches the infinite. None of the other peaks along this north-south ridge zoom to infinity. If you trust only in the equations, a mathematician will believe that there is nothing west of this tall boundary line at the number one. Riemann, although the equations gave him illogical answers, was not defeated by the intractability of the zeta-function. He succeeded in developing a formula to build the missing topography to the west and the resultant geometry of the imaginary landscape did make sense, but the geometry was extraordinarily rigid and what happened to the west of the unit one was fully determined by the landscape built by the coordinate results plotted to the east of the ridge line. It was only by plotting the zeros of the zeta-function that he found a wormhole in how the western landscape unfolded and these zeros were porthole-like 'looking glasses' that telescoped all the information needed to construct the imaginary landscape *in toto*, east, west, north, and south. Counterintuitively, in this imaginary landscape the coordinate locations of all the imaginary numbers where the zeta-function outputs zero revealed everything. To reiterate, it was as you could map a coastline of a continent you were also provided with the GPS points and the height elevation of its highest mountain ranges. The primes create the landscape and the points at sea level, the zeros of the ley line unlock the secrets of the connection. The entire imaginary landscape demonstrated symmetry; Riemann previously knew that the east-west axis of real numbers was a line of symmetry where what happened north of it also was mirrored at the same coordinates to its south. He also determined that his ley-line running north-south through the point at ½ was also an important line of symmetry and if he was correct that all the nontrivial zeta-zeros lay only upon it, then Nature had positioned them so that they were in perfect balance. He inferred correctly that the errors in estimating the number of primes given by his fellow German predecessor Carl Friedrich Gauss and his Prime Number Conjecture could be completely corrected and that Gauss was right in that the higher one counted up the number line, that Gauss's estimate of the number of primes gained precision. Thus, the errors in Gauss's predictions from his prime number estimate formula could be identified and adjusted to acquire the exact number of primes below a given number *N*. Riemann's conjecture, if irrefutable, has strong implications for determining the precise pattern of distribution of how the prime numbers fall out. A pattern that has a hidden order despite being veiled by an appearance of randomness. The nontrivial zeros are hardwired to be fixed on the ½ ley line because of the fundamental if not always obvious interplay between the original starting points [0,1, 2] of the real number line. The ley line is positioned at the midpoint of zero on the x-axis and the number one, the first natural counting number. A span of just one integer, split at its midpoint between 0 and 1, but might this not be a super-diminutive representation of the vast universe between the abyss and unity? We miss not only the fundamental dynamic of the magnanimity of their genesis but arguably the first natural integers represent nullity [0], unity [1] and binary symmetry [2, ½] in a short triad at the very beginning of all number lines. It is the examination of this small interval of origin that perhaps shoots forth a plethora of rhizomic tendrils throughout the rest of mathematics that will prove the truth of Riemann's conjecture; for it somehow mandates that the positioning of the nontrivial zeros lie only within the narrow-plowed furrow anchored at one-half that perpendicularly intersects the x-axis; the seeds planted in this small plot rise up vertically to form nodal peepholes in the form of zeros that bottom out by being dug deep at ½ in the foothills of an undulating mountainous ridge at one; midway from 0 to 1, the low points at the ½ ley line are dug precisely but inversely to when another prime number is needed to continue the upward count of the number line. Remember all numbers can be expressed as the product of prime numbers and at some point no new larger number can materialize without a corresponding new prime and its inversely related nontrivial zero. The nontrivial zeros blooming into nodal existence like the verdant seeds in the bean pods of Jack of fairy tale lore's magical beanstalk providing the structure and fecundity to never stop its kind from growing to the sky and beyond.

Zero, one and two, the latter two numerals are prime and two is the only even prime and the divisor that eliminates any higher even number from being prime. Two is the rival sibling in the even number nest that kills all others from being prime. Zero is the *anti*-prime in that whereas as a prime can only have itself and [1] one as factors, zero can be divided by any number and keep its identity. Although arguably not its scale. It is *curiouser and curiouser,* that Bernhard Riemann was able to 'see through the looking glass' set up with its mirrored pane of glass to see the unfolding of the primes composed of a never-ending string of zeros positioned on a line one above the other at precisely at ½. One divided by two. 0, 1 and 2. Zero, one [unity] are at times pure numbers and at time mathematical concepts like the vacuum of deep space or the vastness of infinity in our cosmos; and like the light rays that illuminate our Universe they too can be waves or particles, these elementary number components can transform themselves from the particulate to the infinite given the circumstances. Two, the only prime number through infinity that is even and not odd. One divided by two is the reciprocal of two divided by one and is a congruent reflection of the inverse nature of the zeta-zeros density and the density of the parading footfall of primes. The density of the prime numbers is approximately the inverse of the nontrivial zeros. It all lines up to be very curious, however! More than odd. And even Alice in Wonderland might still be amazed by the overlooked string of curious coincidences involving nothingness, infinity and the ½ ley line of Riemann. Just imagine that!

By R. Anthony Saritelli January 20, 2025. United States Presidential Inauguration Day.

References:

Du Sautoy, M. *The Music of the Primes. Searching To Solve the Greatest Mystery in Mathematics.* United States. Harper Collins Publishers. 2003.

Rockmore, D. *Stalking the Riemann Hypothesis. The Quest to Find the Hidden Law of Prime Numbers*. Vintage Books. New York. May 2006.

Kitagawa, K. & Revell, T. *The Secret Lives of Numbers: A Hidden History of Math's Unsung Trailblazers.* William Morrow. NY,NY 2023.

Byers, W. *How Mathematicians Think. Using Ambiguity, Contradiction, and Paradox to Create Mathematics.* Princeton University Press. Princeton-Oxford. 2007.